Quantum determinism from quantum general covariance

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Abstract

The requirement of general covariance of quantum field theory (QFT) naturally leads to quantization based on the manifestly covariant De Donder-Weyl formalism. To recover the standard noncovariant formalism without violating covariance, fields need to depend on time in a specific deterministic manner. This deterministic evolution of quantum fields is recognized as a covariant version of the Bohmian hidden-variable interpretation of QFT.

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The reconcilation of quantum theory with general theory of relativity is still an unsolved problem. It is very likely that the successful reconcilation requires a radical reformulation of the basic principles of relativity, or that of quantum theory, or both. One obvious difference between quantum theory and general relativity is that quantum theory, in contrast with general relativity, is an *undeterministic*[1] theory. Most attempts towards the reconcilation start from the assumption that quantum gravity, just as any quantum theory, should also be an undeterministic theory. However, in contrast with this mainstream quantum-undeterministic paradigm, 't Hooft suggests that a fundamental theory that reconciles quantum theory with general relativity should be a *deterministic hidden-variable* theory [2]. As a support for this idea, in this essay we argue that a deterministic hidden-variable formulation of quantum field

[1]Of course, the ideterminism is explicit only in the interpretation of quantum theory related to the problem of measurement. We note that the recent progress in understanding the phenomenon of decoherence through the interaction with the environment shed much light on the problem of measurement in quantum theory, but that this problem is still considered unsolved [1].

theory (QFT) naturally emerges from the requirement that quantum field theory should be general-covariant. In simple terms, restoring one classical property (general covariance) in quantum theory automatically restores another one (determinism). Our discussion is based on recent results first presented by us in [3], which, however, are logically independent of the arguments presented by 't Hooft [2].

Canonical quantization of fields apparently contradicts theory of relativity because the formalism of canonical quantization requires a choice of a special time coordinate. It is known that this fact does not destroy the covariance of QFT with respect to Lorentz transformations [4]. However, what about general coordinate transformations? (In the rest of the paper, by the term "covariant" we mean "general covariant".) QFT can be written in a covariant form by introducing states that are not functions of time, but functionals of an *arbitrary* hypersurface [5, 6, 7, 8, 9, 10]. (The hypersurface is often, but not always, restricted to be timelike.) In this way, there is no preferred foliation of spacetime, so quantization of fields is covariant. However, there is one problem with such a formalism. Without a preferred foliation of spacetime, the notion of a particle in QFT does not have an invariant meaning [11, 12, 13, 14]. Conversely, if a preferred foliation of spacetime is allowed, then the notion of a particle in QFT can be introduced in a local covariant manner [15, 16, 17]. But then the preferred foliation breaks the covariance of the quantization of fields themselves, so, again, the full covariance of the theory is lost.

Is it possible to have *both* quantum fields and particles described in a covariant manner? It *is* possible if a preferred foliation of spacetime is generated *dynamically*. What we need is a dynamical vector quantity R^μ, the direction of which determines the preferred foliation. Since classical field theory is manifestly covariant without a dynamical preferred foliation, this vector should *not* be just another dynamical field that can be treated either as a classical or a quantum field. Instead, it should be a quantity that is inherently related to the quantization formalism itself. Thus, the natural starting point is to consider a scalar quantity of the conventional quantum formalism that can be promoted to a vector by recognizing that the original scalar is actually a time-component of a vector. The most obvious such quantity is the canonical momentum $\pi = \partial\mathcal{L}/\partial(\partial_0\phi)$ (where, for simplicity, $\phi(x)$ is a real scalar field). Clearly, the canonical momentum is a time-component of the momentum vector

$$\pi^\mu = \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}. \tag{1}$$

With the momentum (1), one naturally associates the covariant De Donder-Weyl Hamiltonian (see, e.g., [18, 19] and references therein)

$$\mathcal{H}(\pi^\alpha, \phi) = \pi^\mu\partial_\mu\phi - \mathcal{L}. \tag{2}$$

One can also introduce the covariant De Donder-Weyl Hamilton-Jacobi equation [18, 19]

$$\mathcal{H}\left(\frac{\partial S^\alpha}{\partial\phi}, \phi\right) + \partial_\mu S^\mu = 0, \tag{3}$$

supplemented with the equation that governs the x-dependence of the field

$$\partial^\mu \phi = \frac{\partial S^\mu}{\partial \phi}. \tag{4}$$

In a noncovariant language, equation (4) can be written as two independent equations

$$\partial^0 \phi = \frac{\partial S^0}{\partial \phi}, \quad \partial^i \phi = \frac{\partial S^i}{\partial \phi}. \tag{5}$$

The first equation in (5) represents the "dynamics" and corresponds to an analogous equation in the ordinary noncovariant Hamilton-Jacobi formalism. The second equation in (5) says nothing about the time dependence of the field, so it is merely a "kinematic" equation. However, it is clear that if one requires covariance, then the two equations in (5) are *not* independent. Instead, it is crucial that *if the "kinematic" part of (5) is valid and if covariance is required, then the "dynamic" part of (5) must also be valid.* Another crucial point is the following: In order to recover the ordinary noncovariant Hamilton-Jacobi equation from the covariant Hamilton-Jacobi equation (3), the quantity S^i should be eliminated via the "kinematic" part of (5) [20, 3]. Therefore, the "kinematic" part of (5) *must* be valid.

Now consider quantization. In the conventional noncovariant quantization based on the Schrödinger picture, one replaces the ordinary noncovariant Hamilton-Jacobi equation with the corresponding noncovariant Schrödinger equation. The Schrödinger state $\Psi = Re^{iS/\hbar}$ is desribed by two real functionals R and S. Similarly, in the covariant approach based on the covariant Hamilton-Jacobi equation (3), the quantum state is described by two real *vectors* R^μ and S^μ [3]. (See also [20, 21] for a different approach.) In contrast with S^μ, the vector R^μ does not possess a classical counterpart. Thus, it appears natural to identify R^μ as the vector that dynamically generates the preferred foliation of spacetime [3]. With such a preferred foliation, the correspondence between covariant states and conventional states takes the form

$$S = \int_\Sigma d\Sigma_\mu S^\mu, \quad R = \int_\Sigma d\Sigma_\mu R^\mu, \tag{6}$$

where the integration is taken over a hypersurface Σ that belongs to the dynamically preferred foliation. For other details of the formalism, we refer the reader to [3].

For the subject of this essay, the crucial point is the following. The quantum analog of the covariant Hamilton-Jacobi equation (3) must be compatible with the conventional Schrödinger equation. The conventional Schrödinger equation can be recovered when $R^\mu = (R^0, 0, 0, 0)$. However, just as in the clasical case, the conventional Schrödinger equation can be recovered only if the "kinematic" part of (5) is valid. As we have seen, the requirement of covariance then implies that *the "dynamic" part of (5) must also be valid.* This "dynamic" part says that, in the Schrödinger picture, the field has a *deterministic dependence on time.* On the other hand, in the conventional formulation of the Schrödinger picture of QFT, there is no equation that attributes a deterministic time dependence to the field. Instead, such a time dependence of the field corresponds to the *Bohmian interpretation* of QFT [22, 23, 24, 25, 26, 27]. In the

literature, the Bohmian interpretation is viewed as a deterministic hidden variable theory postulated only for interpretational purposes. Here, the Bohmian interpretation is *not postulated*, but *derived*[2] *from the requirement of covariance.* (Similarly, the Bohmian interpretation of strings can be derived from the world-sheet covariance [28].) This, together with the results of [29, 30] on relativistic first quantization, suggests that it is Bohmian mechanics that might constitute the missing bridge between quantum theory and relativity. The specific theory based on the De Donder-Weyl formalism proposed here may also have measurable consequences based on the fact this theory actually generalizes standard QFT by allowing new types of quantum nonlocalities [3]. As more generic predictions based on a preferred foliation of spacetime that defines a preferred notion of particles we mention the measurable predictions on the Unruh effect [31] and semiclassical gravity [32].

At the end, we note that quantization based on the covariant De Donder-Weyl Hamiltonian leads to covariant quantization not only of matter fields in a fixed curved background (in this case, some of the vectors above should be redefined as vector *densities* [3]), but also of gravity itself [3]. In the case of gravity, all ten components $g_{\mu\nu}$ of the metric tensor are quantized. In contrast with the conventionl noncovariant Wheeler-DeWitt approach to quantum gravity (see, e.g., [33, 34, 35] and references therein), there is no problem of time in the covariant approach. The consistency with the classical noncovariant Hamiltonian constraint is obtained through the use of the covariant Bohmian equations of motion. This is how our covariant deterministic method of quantization resolves some deep conceptual problems of quantum gravity by making quantum gravity more similar to classical gravity.

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